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Blowing and Suction Effects on Free Convection of a Non-Newtonian Fluid Over a Vertical Cone Embedded in a Porous Medium in the Presence of Thermal Radiation and Non-Uniform Heat Generation/Absorption

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Analysis is performed to investigate the effects of thermal radiation and non-uniform heat generation/absorption on free convection boundary layer over a non-isothermal vertical permeable cone in a non-Newtonian power-law fluid saturated porous medium. The governing equations describing the problem are transformed into nonlinear ordinary differential equations using similarity transformation, which are solved numerically. Numerical computations are carried for various values of the parameters on the dimensionless temperature as well as the local Nusselt number. The results show that the thermal radiation parameter, the absolute value of the suction parameter and the heat generation parameter have the effect of depressing the local Nusselt number, whereas the power-law index parameter, the surface temperature exponent parameter, the injection parameter and the heat absorption parameter have the effect of enhancing the local Nusselt number.

Keywords: Vertical Permeable Cone, Non-Newtonian Power-Law Fluids, Non-Uniform Heat Generation/Absorption, Thermal Radiation, Porous Medium.

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1. INTRODUCTION

The investigation of free convection heat transfer in a fluid- saturated porous medium has attracted many investigators due to its applications in many engineering and geophysical applications such as geothermal extraction, storage of nuclear waste material, ground water flows, oil recovery processes, thermal insulation engineering, cooling of electronic components, food processing, casting and welding of manufacturing processes, ceramic processing, the dispersion of chemical contaminants in various processes in the chemical industry. On the other hand, in many physical problems, such as fluids undergoing exothermic or endothermic chemical reactions, the study of the influence of heat generation (absorption) has gained considerable attentions of many authors. There are different forms for the heat generation/absorption. Among of these, the non-uniform heat generation/absorption model which has been used by many investigators.[1–6] The steady free convection flow over a vertical flat plate in the fluid saturated porous media was investigated by Cheng and Minkowycz,[7] Johnson and Cheng,[8] Hung et al.,[9] Mahmood and Merkin,[10] Rees and Pop[11] and Bejan and Khair.[12] The free convection around a vertical cylinder embedded in a porous medium has been analyzed by Minkowycz and Cheng.[13] Cheng et al.[14] analyzed the problem of natural convection about a cone embedded in a porous medium at high Rayleigh numbers based on the boundary layer approximation and the Darcy's law. Pop and Na[15] investigated the natural convection along an isothermal wavy cone embedded in a fluid saturated porous medium. Yih[16] numerically studied the effect of uniform lateral mass flux on natural convection about a cone embedded in a saturated porous medium. Cheng[17] discussed the combined heat and mass transfer by natural convection from truncated cone in saturated porous media with variable wall temperature and concentration. The problem of free convection heat and mass transfer near a wavy cone with constant wall temperature and concentration in a porous medium was examined by Cheng.[18] Chamkha et al.[19] investigated the problem of combined heat and mass transfer by natural convection over a permeable cone embedded in a porous medium in the presence of an external magnetic field and heat generation or absorption. Chamkha[20]

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studied the effect of a magnetic field on free convection from a cone and a wedge in porous media.

All the above studies have dealt with Newtonian fluids. In several applications such as a ceramic processing, enhanced oil recovery, food technology, polymer engineering and liquid composite molding, the fluids involved may exhibit non-Newtonian behavior. Free convection of a non-Newtonian power-law fluid over an isothermal vertical surface in a porous medium was studied numerically by Chen and Chen.[21] Mehta and Rao[22] investigated the buoyancy-induced flow of non-Newtonian fluids in a porous medium over a vertical plate with non-uniform surface heat flux. Chen and Chen[23] presented similarity solution for the problem of natural convection of non-Newtonian power-law fluids around a horizontal surface embedded in a porous medium. Mehta and Rao[24] studied the buoyancy induced flow of non-Newtonian power-law fluids over a non-isothermal horizontal plate using similarity solution. Kumari and Jayanathi[25] analyzed the influence of uniform lateral mass flux on natural convection flow over a vertical cone embedded in a porous medium saturated with non-Newtonian fluid. Yih[26] investigated the uniform lateral mass flux effect on natural convection of non-Newtonian power-law fluids along an isothermal or isoflux vertical cone in a porous medium. Soares et al.[27] examined the influence of temperature dependent viscosity on forced convection heat transfer from a cylinder in cross flow of power-law fluids. Cheng[28] presented a boundary layer analysis about variable viscosity effects on the double-diffusive convection near a vertical truncated cone in a fluid-saturated porous medium with constant wall temperature and concentration. Mahmoud[29] studied the effect of variable viscosity on free convection boundary layer over a vertical cone in a porous medium saturated with non-Newtonian power-law fluid with variable surface heat flux. It is known that the radiation effect is important under many non-isothermal situations. If the entire system involving the polymer extrusion process is placed in thermally controlled environment, the radiation then becomes important. The knowledge of radiation heat transfer in the system can perhaps lead to a desired product with sought characteristics. The influences of radiation on Newtonian and non-Newtonian fluids were investigated by many authors.[30–35] Groşan et al.[36] studied the steady free convection boundary layer over vertical cone embedded in a porous medium filled with a non-Newtonian fluid in the presence of an exponentially decaying internal heat generation. The aim of the present paper is to extend the work of Groşan by including the effects of non-uniform heat generation and thermal radiation on free convection boundary layer over a non-isothermal vertical full permeable cone in a non-Newtonian power-law fluid saturated porous medium. To the best of our knowledge, this problem has not been investigated before despite various applications in engineering processes such as petroleum production.

2. FORMULATION OF THE PROBLEM

Consider the buoyancy-induced flow in a porous medium saturated with a non-Newtonian power law fluid over a vertical full permeable cone. Under the Boussinesq and boundary layer approximations, the governing equations can be written as:

$$\frac{\partial r u}{\partial x} + \frac{\partial r v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u^n}{\partial y} = \frac{\partial}{\partial y}\left(\frac{gK(n)\beta\cos\varphi}{\nu}(T - T_\infty)\right) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{q'''}{\rho c_p} \tag{3}$$

where $r = x\sin\varphi$ is the cone radius, x and y are the Cartesian coordinates along and normal to the generator of the cone. The flow situation being considered here is shown in Figure 1.

u and v are the velocity components along x and y axes, respectively. β is the coefficient of thermal expansion, g is the acceleration due to gravity, q_r is the radiative heat flux, α_m is the effective thermal conductivity, q''' is the internal heat generation, c_p is the specific heat at constant pressure, ρ is the density of the fluid, ν is the kinematic viscosity, T is the fluid temperature, n is the power law index parameter and $K(n)$ is the modified permeability. The fluid is Newtonian for $n = 1$, $n < 1$ and $n > 1$ corresponds to shear thinning and shear thickening fluids, respectively.

The boundary conditions are:

$$\begin{aligned} v = v_w, \quad T_w = T_\infty + Ax^\lambda \ \text{at}\ y = 0 \\ u \to 0, \quad T \to T_\infty \ \text{as}\ y \to \infty \end{aligned} \tag{4}$$

where λ is the surface temperature exponent parameter, A is a positive constant and v_w is the suction velocity (< 0) or injection velocity (> 0).

The radiative heat flux q_r is employed according to Rosseland approximation[31] such that:

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{5}$$

where σ^* is the Stefan-Boltzmann constant and k^* is the mean absorption coefficient. Following Raptis,[32] we

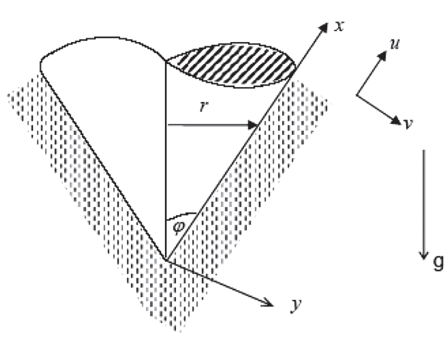

Fig. 1. Physical model and coordinate system.

assume that the temperature difference within the flow are small such that T^4 may be expressed as a linear function of the temperature. Expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, we have

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{6}$$

Now we define the following dimensionless variables :

$$\eta = \sqrt{Ra_x}\left(\frac{y}{x}\right), \quad \psi = \alpha_m r\sqrt{Ra_x} f(\eta), \quad \theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{7}$$

where ψ is the stream function that satisfies the continuity Eq. (1) defined by:

$$ru = \frac{\partial \psi}{\partial y}, \quad rv = -\frac{\partial \psi}{\partial x} \tag{8}$$

and Ra_x is the modified local Rayleigh number, which is defined as:

$$Ra_x = \left(\frac{g\beta K(n)\cos\varphi (T_w - T_\infty)x^n}{\nu \alpha_m}\right)^{1/n} \tag{9}$$

In order that similarity solutions of Eqs. (1)–(3) exist it is assumed that the heat generation q''' is given in the following form:[1]

$$q''' = \frac{\kappa_m}{x^2} Ra_x [\gamma (T_w - T_\infty)e^{-\eta} + \gamma^*(T - T_\infty)] \tag{10}$$

where κ_m is the effective thermal conductivity, γ and γ^* are the coefficient of space-and temperature dependent internal heat generation/absorption, respectively. $\gamma > 0$ and $\gamma^* > 0$ corresponds to internal heat generation, while $\gamma < 0$ and $\gamma^* < 0$ corresponds to internal heat absorption. Using the transformation given in Eq. (7) and using Eqs. (5), (6) and (10) in Eqs. (2) and (3), we obtain the following non-linear ordinary differential equations :

$$(f'^n - \theta)' = 0 \tag{11}$$

$$(1+R)\theta'' - \lambda f'\theta + \left(\frac{3n+\lambda}{2n}\right) f\theta' + (\gamma e^{-\eta} + \gamma^*\theta) = 0 \tag{12}$$

The boundary conditions now become :

$$\begin{aligned} f = f_w, \quad \theta = 1, \ \text{at } \eta = 0 \\ f' \to 0, \quad \theta \to 0, \ \text{as } \eta \to \infty \end{aligned} \tag{13}$$

where $R = (16\sigma^* T_\infty^3)/(3k^*\kappa_m)$ is the radiation parameter, $f_w = 1/\alpha_m (v_w x^{(n-\lambda)/2n}(2n/(\lambda+3n))((Ag\beta K(n)\cos\varphi)/(\nu\alpha_m))^{-1/2n})$ and the primes denote differentiation with respect to η. In order that similarity solutions of Eqs. (1)–(3) exist it is assumed that $v_w \propto x^{\lambda - n/2n}$ It is noted that for $R = 0$ (no radiation interaction) $\gamma^* = 0$ and $\gamma = 1$ Eqs. (11) and (12) reduce to those of Groşan et al.[36]

Important physical parameter of interest in this problem is the local Nusselt number, which is given by :

$$Ra_x^{-1/2} Nu_x = -\theta'(0) \tag{14}$$

3. NUMERICAL SOLUTIONS AND DISCUSSION

The coupled system of non-linear ordinary differential Eqs. (11) and (12) subject to the boundary conditions (13) are solved numerically using the fourth order Runge-Kutta method algorithm with the shooting technique. In order to assess the accuracy of the present numerical method, we compared our numerical results for the local Nusselt number taking $R = 0$, $\gamma^* = 0$ and $\gamma = 0$ in Eqs. (11) and (12) with those obtained by Yih[26] and Groşan et al.[36] in the absence of heat generation, as shown in Table I. It can be seen that the results are in a good agreement.

The results of the numerical computations for temperature profiles for various values of the parameters γ, γ^*, λ, n, R and f_w are illustrated in Figures 2–7. Figure 2 displays the influence of the effect of temperature-dependent heat generation $\gamma^* > 0$ or absorption $\gamma^* < 0$ parameter on the dimensionless temperature θ for both cases $n = 0.8$ and $n = 1.8$. It is shown that as energy absorbed for decreasing the heat absorption ($\gamma^* < 0$, $\gamma < 0$) parameter resulting the temperature to decrease within the boundary layer, whereas the thermal boundary layer thickness increases with increase of the heat generation ($\gamma^* > 0$, $\gamma > 0$) parameter in both cases $n = 0.8$ and $n = 1.8$. The effect of the space-dependent heat generation (absorption) parameter on θ is illustrated in Figure 3 for both cases $n = 0.8$ and $n = 1.8$. It is found that the temperature in the thermal boundary layer decreases with increase in the heat absorption ($\gamma^* < 0$, $\gamma < 0$) parameter, while the temperature profiles increase with increasing the heat generation ($\gamma^* > 0$, $\gamma > 0$) parameter in both $n = 0.8$ and $n = 1.8$. The effects of the surface temperature parameter λ on the temperature distribution are presented in Figure 4. It is observed that an increase in λ leads the temperature profile $\theta(\eta)$ to decrease for both $n = 0.8$ and $n = 1.8$ cases. Figure 5 shows that the thermal boundary layer thickness decreases with the increase of the power-law index parameter n. It is noted that the temperature decreases as the power-law index parameter increases. The effect of the thermal radiation parameter R on $\theta(\eta)$ is shown in Figure 6. The results illustrate that an increase in the thermal radiation parameter leads to an increase in the thickness of thermal boundary layer of the fluid for both $n = 0.8$ and $n = 1.8$ cases.

Table I. Comparison of values of $-\theta'(0)$ for various values of λ and n with $R = 0$, $f_w = 0$ and without heat generation.

	$\lambda = 0$			$\lambda = 1/3$		$\lambda = 1/2$	
n	Yih[26]	Groşan et al.[36]	Present work	Groşan et al.[36]	Present work	Groşan et al.[36]	Present work
0.5	0.6522	0.6527	0.6527	0.8172	0.8166	0.8828	0.8827
0.8	0.7339	0.7340	0.7339	0.8884	0.8884	0.9574	0.9574
1	0.7686	0.7686	0.7686	0.9211	0.9210	0.9897	0.9896
1.5	0.8233	0.8233	0.8233	0.9729	0.9729	1.0409	1.0409
2	0.8552	0.8552	0.8552	1.0033	1.0034	1.0710	1.0710

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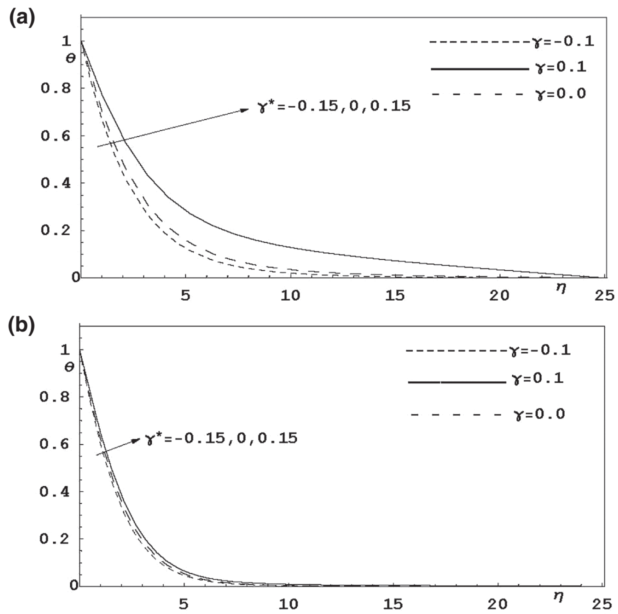

Fig. 2. (a) Temperature profiles for various values of γ^* with $n = 0.8$, $R = 5$ and $\lambda = 0.5$. (b) Temperature profiles for various values of γ^* with $n = 1.8$, $R = 5$ and $\lambda = 0.5$.

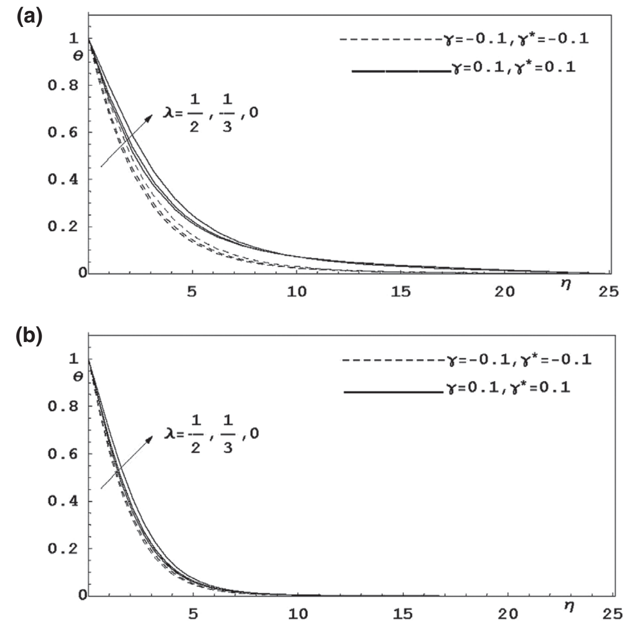

Fig. 4. (a) Temperature profiles for various values of λ with $n = 0.8$, and $R = 5$. (b) Temperature profiles for various values of λ with $n = 1.8$, and $R = 5$.

Also, increasing the values of R is to enhance the temperature profile through the boundary layer region. This is due to the fact that higher values of the radiation parameter R lead to a decrease in Rosseland mean absorption coefficient k^* for a given κ and T_∞. Thus, we can deduce from Eqs. (3) and (5) that the divergence of the radiative heat flux $\partial q_r/\partial y$ rises as k^* decreases and consequently the fluid temperature increases. From Figures 2–6, it can be seen that the presence of heat generation effects ($\gamma > 0$, $\gamma^* > 0$) has a tendency to increase the fluid temperature, causing the thermal boundary layer thickness to increase, on contrary the thermal boundary layer thickness decreases for the heat absorption effect ($\gamma^* < 0$, $\gamma < 0$). Figure 7 displays the effect of the injection parameter ($f_w > 0$) and the suction parameter ($f_w < 0$) on the temperature θ. It is noticed that the temperature increase as the absolute value of the suction parameter increases for both $n = 0.8$ and $n = 1.8$. This emphasize the usual fact that suction stabilizes the boundary layer grows. The injection parameter has exactly the opposite effect.

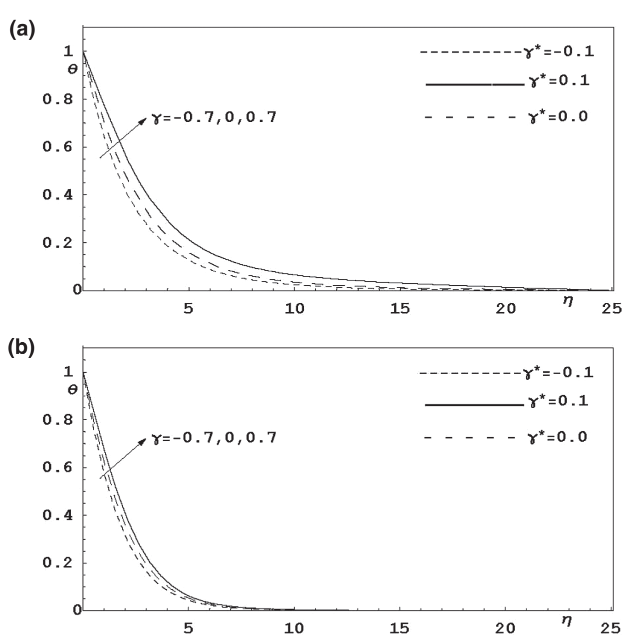

Fig. 3. (a) Temperature profiles for various values of γ with $n = 0.8$, $R = 5$ and $\lambda = 0.5$. (b) Temperature profiles for various values of γ with $n = 1.8$, $R = 5$ and $\lambda = 0.5$.

The influence of various parameters n, λ, R, f_w, γ and γ^* on the local Nusselt number in terms of $-\theta'(0)$ are illustrated in Table II. From this table, it is observed that the local Nusselt number increases with the increasing values of λ, this is due to the fact that as λ increases the

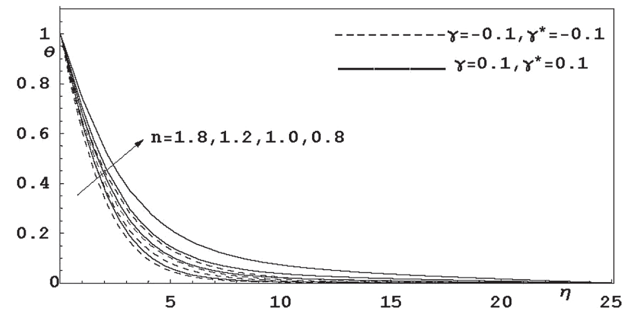

Fig. 5. Temperature profiles for various values of n with $R = 1$, $\lambda = 1/3$ and $\alpha = 0.2$.

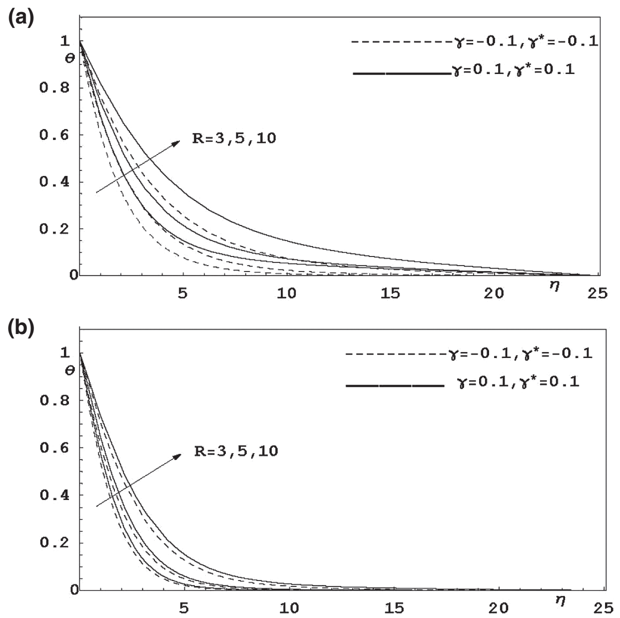

Fig. 6. (a) Temperature profiles for various values of R with $n = 0.8$, and $\lambda = 0.5$. (b) Temperature profiles for various values of R with $n = 1.8$, and $\lambda = 0.5$.

temperature at the wall increases, which causes higher heat transfer, i.e., increasing the local Nusselt number. Also, it is noted that increasing values of n or the absolute value of the heat absorption sink parameter ($\gamma < 0,\ \gamma^* < 0$) leads to increasing the values of the local Nusselt number. Moreover, the local Nusselt number decreases as the heat generation source parameter ($\gamma > 0,\ \gamma^* > 0$) or the thermal radiation parameter increases. However, it is shown that the values of the local Nusselt number in the presence of heat generation source ($\gamma > 0,\ \gamma^* > 0$) are greater than in the presence of heat absorption source ($\gamma < 0,\ \gamma^* < 0$).

Table II. Values of $-\theta'(0)$ with $f_w = 0$ for different values of n, λ, R, γ and γ^*.

				$n = 0.8$	$n = 1.8$
λ	R	γ	γ^*	$-\theta'(0)$	$-\theta'(0)$
0.0	5	0.1	0.1	0.194283	0.301637
1/3	5	0.1	0.1	0.254124	0.365169
0.5	5	0.1	0.1	0.280391	0.393952
0.0	5	−0.1	−0.1	0.296658	0.371622
1/3	5	−0.1	−0.1	0.349939	0.42904
0.5	5	−0.1	−0.1	0.374337	0.455444
0.5	3	0.1	0.1	0.353488	0.484302
0.5	5	0.1	0.1	0.280391	0.393952
0.5	10	0.1	0.1	0.189311	0.28533
0.5	3	−0.1	−0.1	0.471844	0.562928
0.5	5	−0.1	−0.1	0.374343	0.455444
0.5	10	−0.1	−0.1	0.260812	0.329776
0.5	5	−0.7	−0.1	0.445636	0.524956
0.5	5	−0.1	−0.1	0.374337	0.455444
0.5	5	0	0.1	0.291858	0.405698
0.5	5	0.1	0.1	0.280391	0.393952
0.5	5	0.7	0.1	0.211732	0.323709
0.5	5	−0.1	−0.15	0.38692	0.464378
0.5	5	−0.1	−0.1	0.374337	0.455444
0.5	5	0.1	0	0.32236	0.41371
0.5	5	0.1	0.1	0.280391	0.393952
0.5	5	0.1	0.15	0.242003	0.382632

Table III. values of $-\theta'(0)$ for different values of γ, γ^* and f_w with $\lambda = 1/3$ and $R = 5$.

			$n = 0.8$	$n = 1.8$
γ	γ^*	f_w	$-\theta'(0)$	$-\theta'(0)$
0.1	0.1	0.1	0.269634	0.380675
0.1	0.1	−0.1	0.239242	0.350024
0.1	0.1	0.3	0.30278	0.412739
0.1	0.1	−0.3	0.211221	0.320838
0.1	0.1	0.5	0.338204	0.446153
0.1	0.1	−0.5	0.185725	0.29316
0.1	0.1	0.7	0.375913	0.480852
0.1	0.1	−0.7	0.160635	0.267036
−0.1	−0.1	0.1	0.364976	0.44444
−0.1	−0.1	−0.1	0.335408	0.413973
−0.1	−0.1	0.3	0.396484	0.476241
−0.1	−0.1	−0.3	0.307711	0.384876
−0.1	−0.1	0.5	0.429881	0.5093003
−0.1	−0.1	−0.5	0.282007	0.357187
−0.1	−0.1	0.7	0.465118	0.54345
−0.1	−0.1	−0.7	0.258161	0.330937

4. CONCLUSIONS

The problem of free convection of non-Newtonian power-law fluids on a non-isothermal vertical full cone embedded in a porous medium is investigated taking into account non-uniform heat generation (absorption) and thermal radiation. The transformed nonlinear ordinary differential equations were solved numerically using the fourth-order Runge-Kutta scheme with the shooting method. It was found that the local Nusselt number increases with increasing the surface temperature parameter or the power-law index parameter or the absolute value of the heat absorption sink parameter. Also, it was found that the local Nusselt number decreases with the increase in the thermal radiation parameter or the heat generation source parameter.

References

1. R. Tsai, K. H. Huang, and J. S. Huang, *Int. Comm. Heat Mass Transfer* 35, 1340 (**2008**).
2. M. S. Abel, J. Tawade, and Nandeppanavar, *Int. J. Non-Linear Mech.* 44, 990 (**2009**).
3. D. Pal and H. Mondal, *Commun. Non-linear Sci. Numer. Simulat.* 15, 1533 (**2010**).
4. D. Pal, *Commun. Non-linear Sci. Numer. Simulat.* 16, 1890 (**2011**).
5. L. Zheng, L. Wang, and X. Zhang, *Commun. Non-linear Sci. Numer. Simulat.* 16, 731 (**2011**).
6. B. J. Gireesha, G. K. Ramesh, M. S. Abel, and C. S. Bagewadi, *Int. J. Multiphas flow In Press* (**2011**).
7. P. Cheng and W. J. Minkowycz, *J. Geophys. Res.* 82, 2040 (**1977**).
8. C. H. Johnson and P. Cheng, *Int. J. Heat Mass Transfer* 21, 709 (**1978**).

9. C.-I. Hung, C.-H. Chen, and C.-B. Chen, *Int. J. Eng. Sci.* 37, 477 **(1999)**.
10. T. Mahmood and J. H. Merkin, *Transport in Porous Media* 32, 285 **(1998)**.
11. D. A. S. Rees and I. Pop, *Int. J. Heat Mass Transfer* 34, 2565 **(2000)**.
12. A. Bejan and K. R. Khair, *Int. J. Heat Mass Transfer* 28, 909 **(1985)**.
13. W. J. Minkowycz and P. Cheng, *Letters Heat Mass Transfer* 9, 159 **(1982)**.
14. P. Cheng, T. T. Le, and I. Pop, *Int. Commun. Heat Mass Transfer* 12, 705 **(1985)**.
15. I. Pop and T. Y. Na, *Int. Comm. Heat Mass Transfer* 21, 891 **(1994)**.
16. K. A. Yih, *Int. Comm. Heat Mass Transfer* 24, 1195 **(1997)**.
17. C. Y. Cheng, *Int. Comm. Heat Mass Transfer* 27, 537 **(2000)**.
18. C. Y. Cheng, *Mech. Res. Commun.* 27, 613 **(2000)**.
19. A. J. Chamkha and M. M. A. Quadri, *Heat and Mass Transfer* 38, 487 **(2002)**.
20. A. J. Chamkha, *Int. Commun. Heat Mass Transfer* 23, 875 **(1996)**.
21. H. T. Chen and C. K. Chen, *ASME J. Heat Transfer* 110, 257 **(1988)**.
22. K. N. Mehta and K. N. Rao, *Int. J. Eng. Sci.* 32, 297 **(1994)**.
23. H. T. Chen and C. K. Chen, *J. Energy Resources Technology* 109, 119 **(1987)**.
24. K. N. Mehta and K. N. Rao, *Int. J. Eng. Sci.* 32, 521 **(1994)**.
25. M. Kumari and S. Jayanathi, *Journal of Porous Media* 8, 73 **(2005)**.
26. K. A. Yih, *Int. Comm. Heat Mass Transfer* 25, 959 **(1998)**.
27. A. A. Soares, J. M. Ferreira, L. Caramelo, J. Anacleto, and R. P. Chhabra, *Int. J. Heat Mass Transfer* 53, 4728 **(2010)**.
28. C. Y. Cheng, *Applied Mathematics and Computation* 212, 185 **(2009)**.
29. M. A. A. Mahmoud, *Eur. Phys. J. Plus* 126, 5 **(2011)**.
30. A. Raptis, *Int. J. Heat Mass Transfer* 41, 2865 **(1998)**.
31. A. Raptis, *Int. Comm. Heat Mass Transfer* 26, 889 **(1999)**.
32. A. Raptis, C. Perdikis, and H. S. Takhar, *Appl. Math. Compt.* 153, 645 **(2004)**.
33. M. A. A. Mahmoud, *Physica A* 375, 401 **(2007)**.
34. M. S. Abel and N. Mahesha, *Applied Mathematical Modelling* 32, 1965 **(2008)**.
35. S. Mukhopadhyay, *Int. J. Heat Mass Transfer* 52, 3261 **(2009)**.
36. T. Groşan, A. Postenlnicu, and I. Pop, *Technische Mechanik* 24, 91 **(2004)**.

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